8/4/2023

Relations and functions

Cartesian product of sets

$$A \times B = \{(a,b) : a \in A ; b \in B\}$$

Ex:- $A = \{a, b\}$

$B = \{1, 2, 3\}$

$A \times B \{(a,1)\ (a,2)\ (a,3)\ (b,1)\ (b,2)(b,3)\}$

$(a_1, b_1) = (a_2, b_2)$

$\Rightarrow a_1 = a_2,\ b_1 = b_2$

Exercise -2.1

1. $\left(\frac{x}{3}+1, y-\frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$, find the volume of x and y

$a_1 = a_2$

$\frac{x}{3}+1 \quad \frac{5}{3}$

$\frac{x+3}{3} = \frac{5}{3} \Rightarrow \frac{x}{3} + \frac{3}{3} = \frac{1}{1} \Rightarrow \frac{5-3}{3} \Rightarrow \frac{2}{3}$

$x = 5-3$

$x = 2$

$b_1 = b_2$

$\frac{3y-2}{3} = \frac{1}{3}$

$3y = 1+2$

$y = \frac{3}{3} = 1$

2.

$n(A) = 3$
$n(B) = 3$
$n(A \times B) = 3 \times 3$
$= 9$ (Ans)

3.

$G = \{7, 8\}$
$H = \{5, 4, 2\}$

$n(G) = 2$
$n(H) = 3$
$n(G \times H) = 6$

$n(G \times H)$ (7,5), (7,4) (7,2) (8,6) (8,4) (8,2)
$n(H \times G)$ (5,7), (4,7) (2,7) (5,8) (4,8) (2,8)

$n(H \times G) = 6$

4.

(i) false
(ii) True
(iii) True

5. $A = \{-1, 1\}$

$A \times A \times A$

$\{-1, 1\} \times \{-1, 1\} \times \{-1, 1\}$

5. $A = \{-1, 1\}$

$A \times A \times A$

$A \times A \{(-1,-1), (-1,1) (1,-1), \{1,1\}, \{-1,1\}$

$A \times A \times A \{(-1,-1,-1), (-1,-1,1), (-1,1,-1) (-1,1,1)\}$

6. $A \times B = \{(a,x), (a,y), (b,x), (b,y)\}$ Find A and B

$A \times B = \{(a,x) (a,y) (b,x) (b,y)\}$

$A = \{a, b\}$ $B = \{x, y\}$

7.

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$A \times (B \cap C) = (A \times B) \cap (A \times C$

$B \cap C = \phi$

$A \times (B \cap C) = A \times \phi = \phi$

$A \times B = \{(1,1) (1,2) (1,3) (1,4) (2,1) (2,2) (2,3) (2,4)\}$

$A \times C = \{(1,5) (1,6) (2,5) (2,6)\}$

$A \times B \cap (C \times C) = \phi$

L.H.S. = R.H.S.

(iii) $A \times C = \{(1,5), (1,6) (2,5) (2,6)\}$

$B \times D = \{(1,5) (1,6) (1,7) (1,8) (2,5) (2,6) (2,7) (2,8) (3,5) (3,$
$(3,7) (3,8) (4,5) (4,6) (4,7) (4,8)\}$

$(A \times C) \subset (B \times D)$

8. $A = \{1, 2\}$
$B = \{3, 4\}$

$A \times B = \{(1,3)\ (1,4)\ (2,3)\ (2,4)\}$
$n(A \times B) = 4$
$= 2^n$
$= 2^4$
$= 16$

9. $n(A) = 3$ $n(B) = 2$
$A = \{x, y, z\}$ $B = \{1, 2\}$

$n(A \times A) = 9$
$A = \{-1, 0, 1\}$

10. $n(A \times A) = 9$ $n(A) = 3 \times 3$ $n(A) = 3$
$A = \{-1, 0, 1\}$
$A \times A = \{(-1,-1), (-1,0)\ (-1,1)\ (0,-1)\ (0,0)\ (0,1)\ (1,-1)\ (1,0)\ (1,1)\}$

Remaining element - $\{(-1,-1), (-1,1), (0,-1), (0,0), (1,-1), (1,0), (1,1)\}$

Exercise - 2.2

1.

$A = \{1, 2, 3 \ldots\ldots\ldots 14\}$

$R : A \to A$

$R = \{(x, y) : 3x - y = 0, x, y \in A\}$

$y = 3x$

$R = \{(1,3)\ (2,6)\ (3,9)\ (4,12)\}$

Domain $= \{1, 2, 3, 4\}$

Range $= \{3, 6, 9, 12\}$

Codomain $= \{1, 2, 3 \ldots\ldots 14\}$

2. $R = \{(x, y : y = x + 5\}$

x is a natural no. less than 4, $x, y \in N$

ii) $R = \{(1,6), (2,7), (3,8)\}$

(iii) Domain $= \{1, 2, 3\}$

Range $(R) = \{6, 7, 8\}$

3.

$A = \{1, 2, 3, 5\}$

$B = \{4, 6, 9\}$

$R = \{(x, y) :$ the difference b/w x & y is odd; $x \in A, y \in B\}$

$R = \{(1,4)\ (1,6)\ (2,9)\ (3,4)\ (3,6)\ (5,4)\ (5,6)\}$

4.

(i) in set builder form

$P = \{5, 6, 7\}$ $Q = \{3, 4, 5\}$

$R = \{(x, y) : y = x - 2 ; x \in P\}$ or $R = \{(x, y) : y\ x - 2$ where $x = 5, 6, 7\}$

(ii) Roster form

$R = \{(5, 3)\ (6, 4)\ (7, 5)\}$

Domain of $R = \{5, 6, 7\}$

Range of $R = \{3, 4, 5\}$

5. Let - - - - - - - - - - - - - by

(i) write R in Roster form

given:- $\{1, 2, 3, 4, 6\}$ are relation $R = \{(a, b) ; a, b \in A$, b is exact divisible by a$\}$

$R = \{(1, 1)\ (1, 2)\ (1, 3)\ (1, 4)\ (1, 6)\ (2, 2)\ (2, 4)\ (2, 6)\ (3, 3)\ (3, 6)(4, 4)\ (6, 6)\}$

(ii) Find the domain of R

Domain of $R = \{1, 2, 3, 4, 6\}$

(iii) Find the range of R

Range of $R = \{1, 2, 3, 4, 6\}$

6. Determine - - - - - - - - - - - - - -

Given:-

Relation $R = \{(x, x+5) : x \in \{0, 1, 2, 3, 4, 5\}\}$

$R = \{(0, 5)\ (1, 6)\ (2, 7)\ (3, 8)\ (4, 9)\ (5, 10)\}$

Domain of $R = \{0, 1, 2, 3, 4, 5\}$

Range of $R = \{5, 6, 7, 8, 9, 10\}$

7. Write - - - - - - - - - - - - - - - - form

Relation $R = \{(x, x^3) : x$ is a prime no. less than $10\}$
The prime numbers less than 10 are 2, 3, 5, 7
So,
$R = \{(2,8)\ (3,27)\ (5,125)\ (7,343)\}$

8. Let - - - - - - - - - - - - - - - A to B
Given:- $A = \{x, y, z\}$ $B = \{1, 2\}$
$A \times B = \{(x,1)\ (x,2)\ (y,1)\ (y,2)\ (z,1)\ (z,2)\}$
As $n(A \times B) = 6$ the no. of subsets of $A \times B = 2^6$

9. Let - - - - - - - - - - - - - - - - - R
Given,
Relation $R = \{(a,b) : a, b \in Z, a-b$ is an integer$\}$
Domain of $R = Z$
Range of $R = Z$

5/May/2023

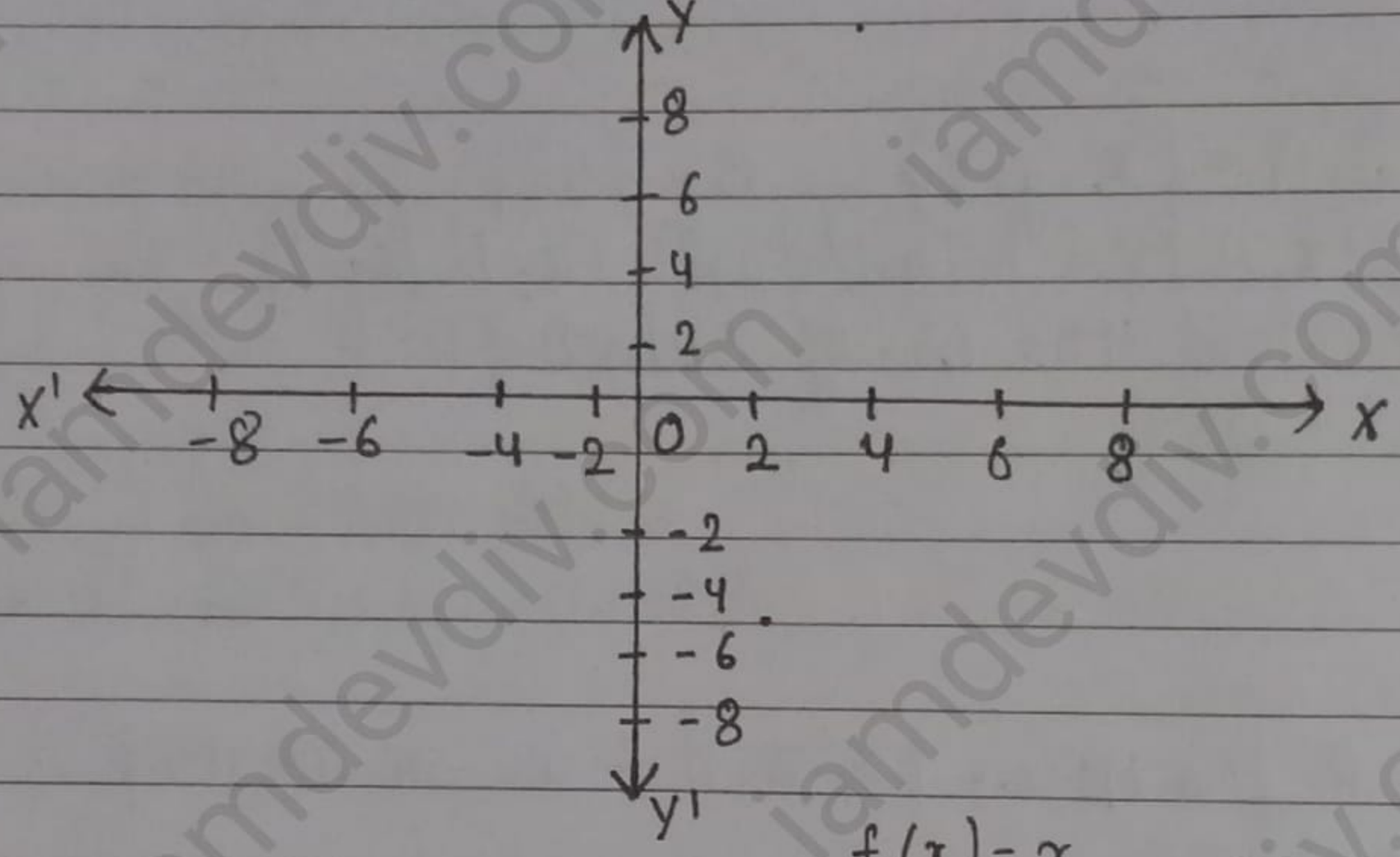
→ Some functions and their graph

(i) Identity function:-

A real value function $f: R \to R$ by $y = f(x) = x$ $\forall$ (for all) $x \in R$.

Such a function is called identity function

It's domain and range are 'R'.



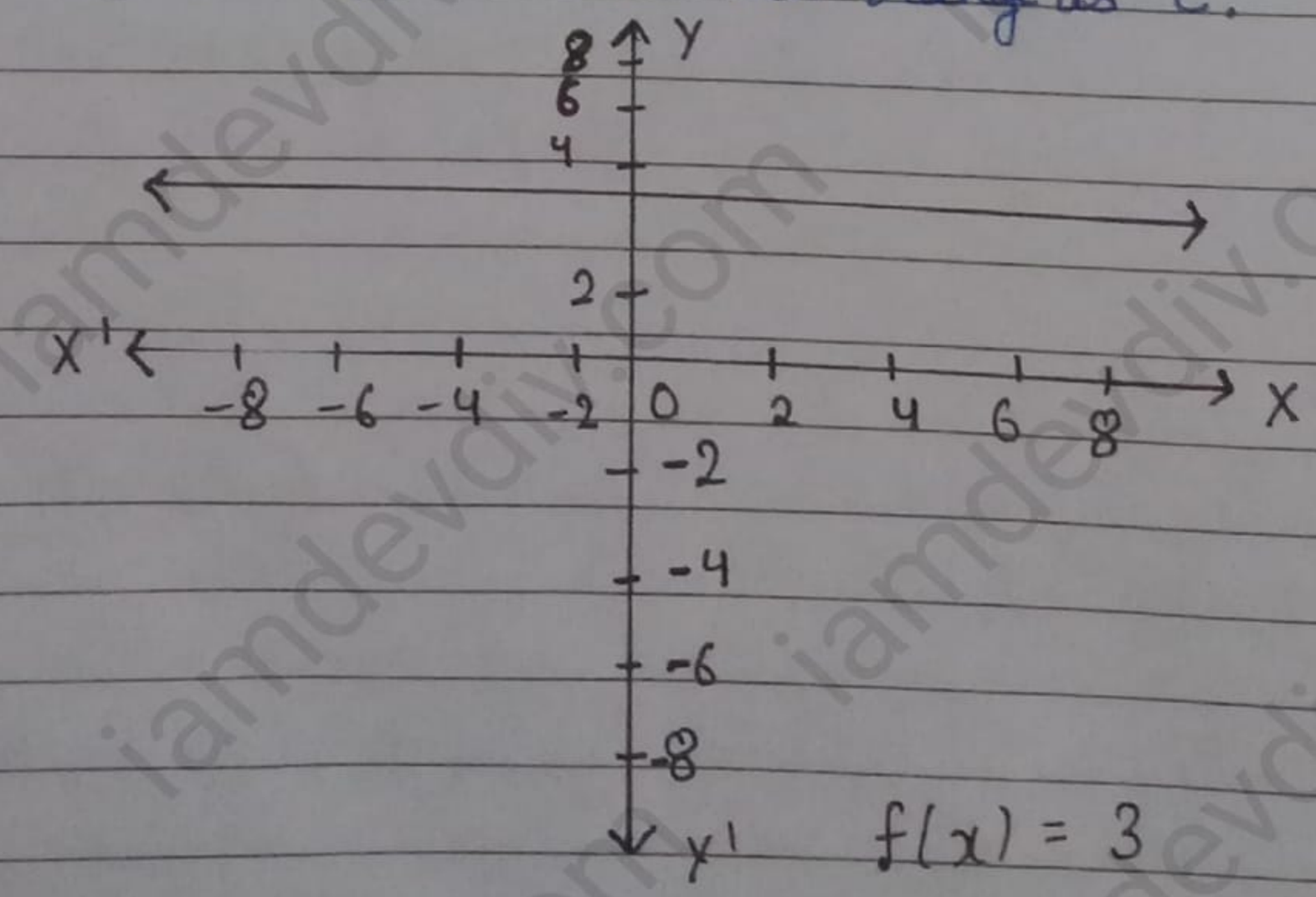
$f(x) = x$

(ii) Constant function:-

A function $f: R \to R$ by $f(x) = C$ $\forall$ $x \in R$.

Such a function is called Constant function.

It's domain is 'R' and ~~its~~ range is 'C'.



$f(x) = 3$

(iii) Polynomial function:-

A function $f: R \to R$ is said to be polynomial function if for each x in R, $y = f(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$, wherein is a non-negative integer and $a_0, a_1, a_2, \ldots, a_n \in R$

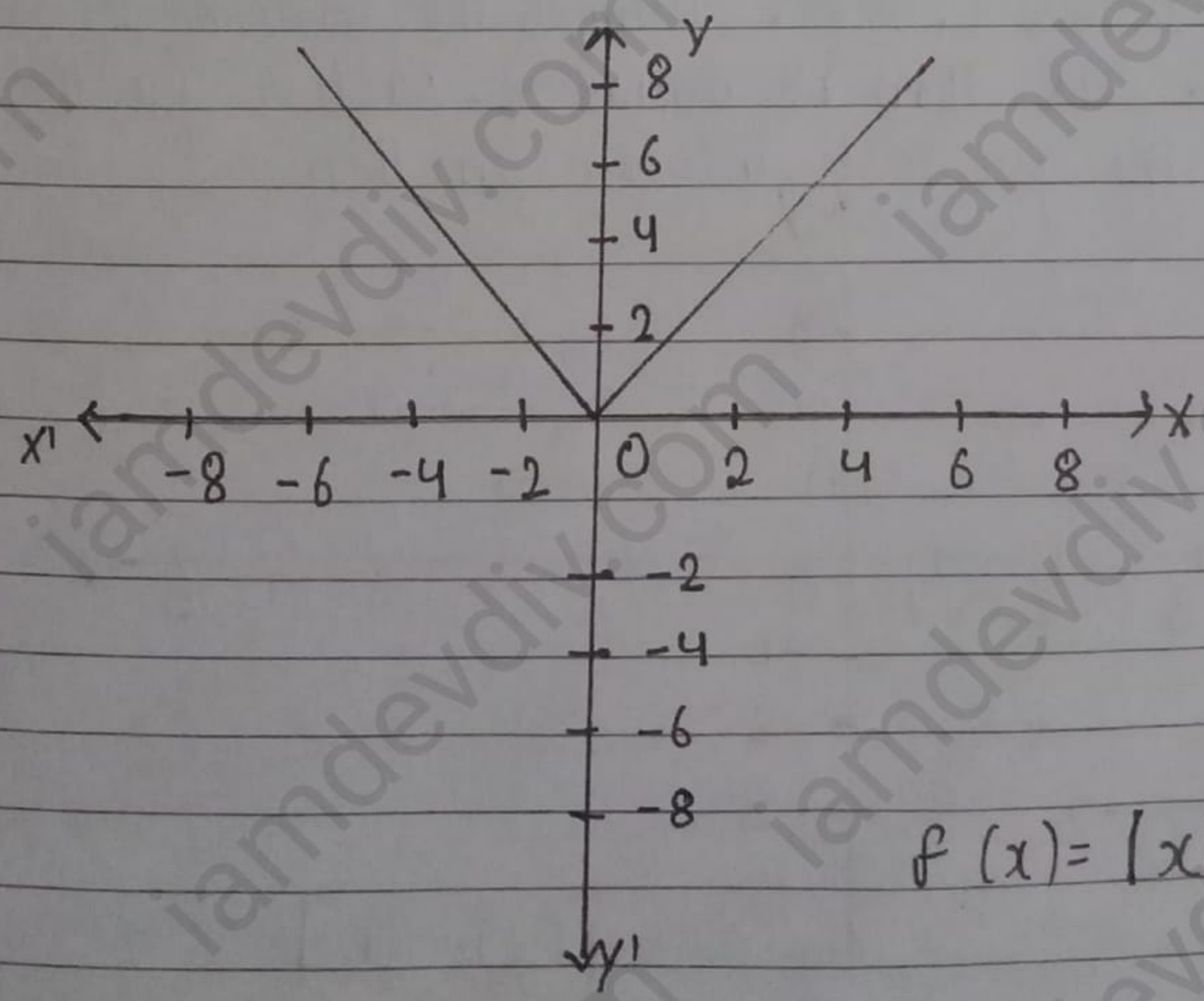
(iv) Rational function:-

A function of the type $\frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are polynomial function of x defined in a domain; where $g(x) \neq 0$

(v) The Modulus function

The function $f: R \to R$ defined by $f(x) = |x|$ for each $x \in R$ is called modulus function.

$$f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

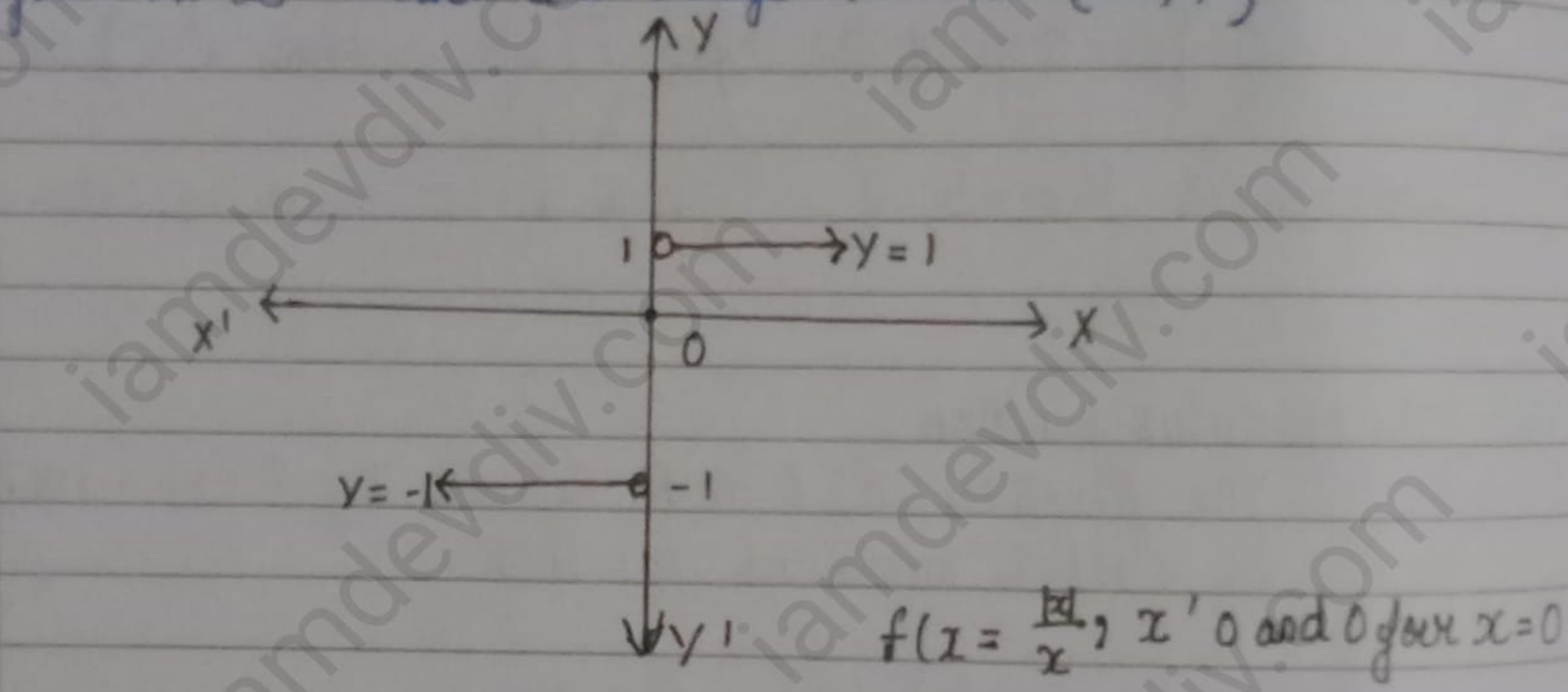


$f(x) = |x|$

(vi) Signum function:- The function $f: R \to R$ defined by

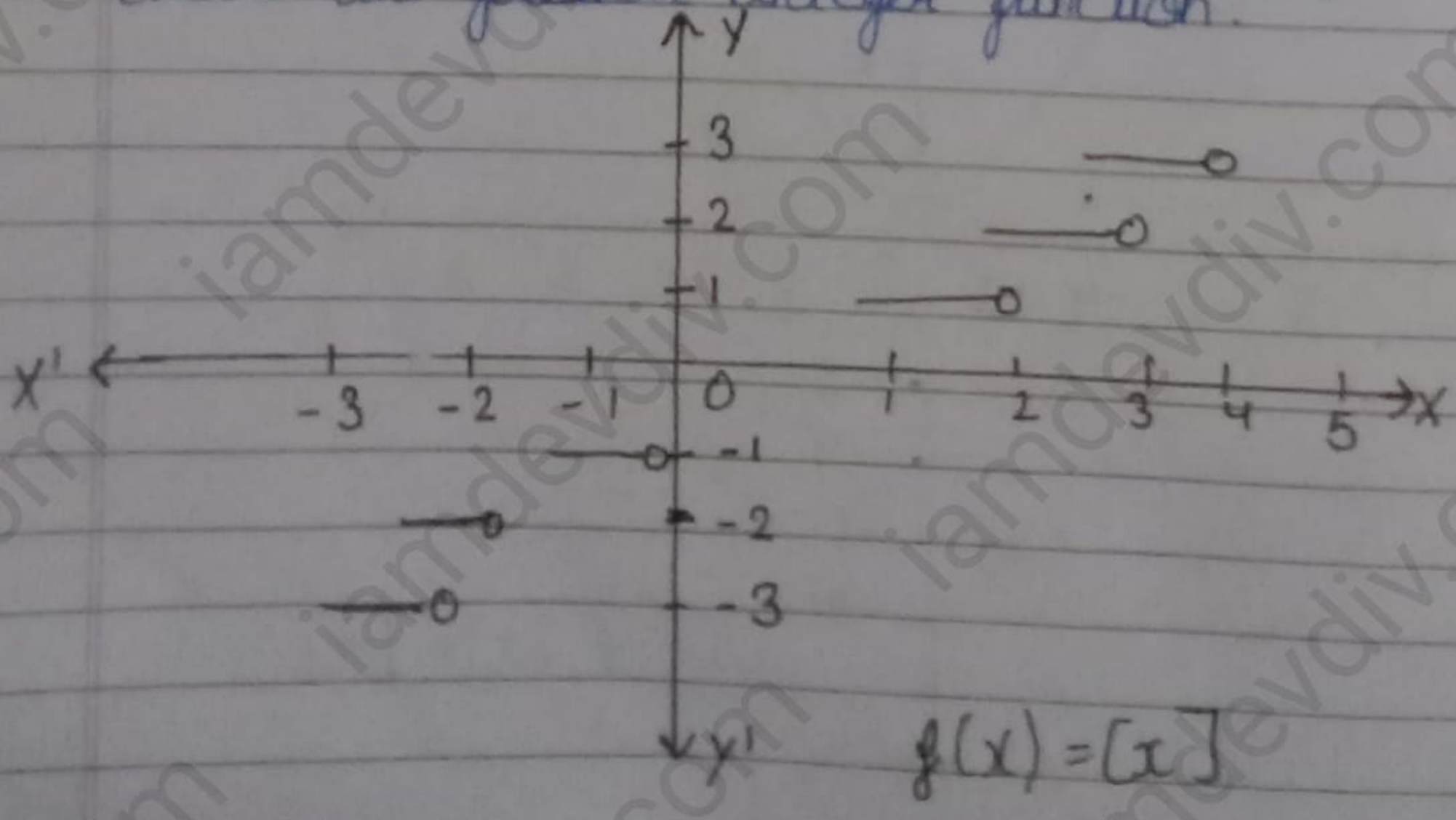
$$f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$

is called the signum function. The domain of the signum function is R and the range the set $\{-1, 0, 1\}$



$f(x) = \frac{|x|}{x}$, $x \neq 0$ and 0 for $x = 0$

(vii) Greatest integer function:- The function $f: R \to R$ defined by $(fx) = [x]$, $x \in R$ assumes the value of the greatest integer, less than or equal x. such a function is called the greatest integer function.



$f(x) = [x]$

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Exercise - 2.3

1. which . . . . . range.

(i) {(2,1), (5,1), (8,1) (11,1) (14,1) (17,1)}
It is a function because it's domain are different and range is similar

(ii) {(2,1), (4,2) (6,3), (8,4), (10,5), (12,6), (14,7)}
It is a function because it's range is in the order.

(iii) {(1,3), (1,5), (2,5)}
It is not a function because it's domain are similar

2. find . . . . . value of
$f(x) = 2x - 5$

(i) $f(0)$

$f(0) = 2(0) - 5$
$= 0 - 5$
$= -5$

(ii) $f(7)$

$f(7) = 2(7) - 5$
$= 14 - 5$
$= 9$

(iii) $f(-3)$

$$f(-3) = 2(-3) - 5$$
$$= -6 - 5$$
$$= -11$$

4. The - - - - - - - - - - - - - -

$$t(c) = \frac{9c}{5} + 32$$

(i) $t(0)$

$$t(0) = \frac{9(0)}{5} + 32 = \frac{0}{5} + 32 = 32 \text{ (Ans)}$$

~~$= \frac{0+32}{5} = 0 + \frac{32}{5}^{6.4} = 6.4$~~

(ii) $t(28)$

$$t(28) = \frac{9(28)}{5} + \frac{32}{1}$$

~~$= \frac{282}{5}^{56.9} = 56.4$~~

$$= \frac{252 + 160}{5}$$

$$= \frac{412}{5} \text{ (Ans)}$$

(iii) $t(-10)$

$$t(-10) = \frac{9(-10)}{5}^{2} + 32$$

$$= -\frac{18}{5} + 32$$

$$= -\frac{58}{5}$$

$$= 14 \text{ (Ans)}$$

(iv) The value of C when $t(C) = 212$

$$t(c) = 212$$

$$\frac{9c}{5} + 32 = 212$$

$$\frac{9c}{5} = 212 - 32$$

$$\frac{9c}{5} = 180$$

$$9c = 180 \times 5$$

$$c = \frac{2980 \times 5}{9}$$

$$c = 100 \text{ (Ans)}$$

5. find - - - - - - functions

(ii) $f(x) = 2 - 3x, x \in R, x > 0$

$R = (-\infty, 2)$

(iii) $f(x) = x^x$, is a real number

$R = R$

2. Find - - - - - - - - real functions.

a.) $fx = -|x|$

Domain = R

Range = $(-\infty, 0)$

b.) $f(x) \sqrt{9 - x^2}$

Domain = $\{x : -3 \leq x \leq 3\}$

Range = $\{x : 0 \leq x \leq 3\}$

Miscellaneous Exercise

1. The relation f is defined by $f(x) = \begin{cases} x^2, 0 \leq x \leq 3 \\ 3x, 3 \leq x \leq 10 \end{cases}$

The relation g is defined by $g(x) = \begin{cases} x^2, 0 \leq x \leq 2 \\ 3x, 2 \leq x \leq 10 \end{cases}$

at $x = 3$

$f(3) = x^2 = (3)^2 = 9$

$f(3) = 3x = 3 \times 3 = 9$

f has a unique image

F is a function

$g(2) = x^2 = (2)^2 = 4$

$g(2) = 3x$

$= 3(2) = 6$

$= g$ is not a function.

3. Find - - - - - - - - - - - -

$$f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$$

$x^2 - 8x + 12 = 0$

$x^2 - 6x - 2x + 12 = 0$

$x(x-6) - 2(x-6) = 0$

$x - 6 = 0 \quad x - 2 = 0$

$x = 6 \quad x = 2$

Domain = $R = \{2, 6\}$

4. Find - - - - - - - - - - - - - $f(x) = \sqrt{(x-1)}$

$f(x)$ $\sqrt{x-1}$

$y = \sqrt{x-1}$

for real y

$x - 1 \geq 0$

$x \geq 1$

Domain $= (1, \infty)$

Range $y = \sqrt{x-1}$

$y^2 = \sqrt{x-1}$

Range $= [0, \infty]$

7. Let $f, g : R \to R$ - - - - - - - - - - - - - - - $\frac{f}{g}$

$f, g : R \to R$

$f(x) = x + 1$

$g(x) = 2x - 3$

$(f+g)(x) = f(x) + g(x)$

$= x + 1 + 2x - 3$

$= 3x - 2$ (Ans)

$(f-g) = f(x) - g(x)$
$= x + 1 - 2x + 3$
$= -x + 4$ (Ans)

Q8- Let . . . . . a, b

$f = \{(1,1)\ (2,3)\ (0,-1)\ (-1,-3)\}$
$y = f(x) = ax + b$ - (1)
$= (0,-1)$
$x\ \ y$

$x = 0 \qquad y = -1$
$y = ax + b$
$-1 = a \times 0 + b$
$-1 = 0 + b$
$b = -1$
$(1,1)$
$x\ \ y$

Putting these values in eq (1)
$1 = a \times 1 + b$
$1 = a - 1$
$1 + 1 = a$
$a = 2$
$b = -1$ (Ans)

Ex

$(a,a) \notin R \Rightarrow a \neq a^2$

which is

Ques 9 - Let - - - - - - - - - - - - - - true?

(i) $(a,a) \in R$, for all $a \in N$

$(a,a) \in R$

$a = a^2$

which is true only for 1 & not for others

(iii) $(a,b) \in R$, implies $(b,a) \in R$

$a = 4 \quad b = 2$

$(a,b) \in R$

$a = b^2$

$4 = 2^2$

$4 = 4$

(iii) $(a,b) \in R$, $(b,c) \in R$ implies $(a,c) \in R$

Let $a = 4$, $b = 7$ and $c = 1$

then $(4,2) \in R$, $(2,1) \in R$ implies $(4,1)$

4 has two images 2 & 1

∴ It is not true

12. Let A . . . - - - - - - - - - - f

$A = \{9, 10, 11, 12, 13\}$

$f : A \to N$

$f(n) =$ highest prime factor of n

Range $(f) = \{3, 5, 11, 13\}$

10.

$A = \{1, 2, 3, 4\}$ $B = \{1, 5, 9, 11, 15, 16\}$

$f = \{(1, 5)\ (2, 9)\ (3, 1)\ (4, 5)\ (2, 1)\}$

(i) f is a function because A X B follows an order
(ii) f is not a function as 2 has two images

Ques2 - $f(x) = x^2$

$f(1.1) = (1.1)^2 = 1.21$ $f(1) = (1)^2 = 1$

$$\frac{f(1.1) - f(1)}{(1.1) - (1)} = \frac{1.21 - 1}{1.1 - 1} = \frac{0.21}{0.1} = \frac{21}{10} = 2.1$$

Ques5- $f(x) = (x-1)$
Domain = R Range = All positive Real no.

Ques6- $f = \{(x, \frac{x^2}{1+x^2}) : x \in R\}$

We know that $x \in R$

$$x^2 \geq 0$$

Then $x^2 + 1 \geq x^2$

$$1 \geq \frac{x^2}{(x^2+1)}$$

The range of F = [0, 1)

Quest- $f = \{(ab, a+b) : a, b \in z\}$

$2, 6, -2, -6 \in Z$

$f = (2 \times 6, 2+6), (-2 \times -6, -2, -6)$

$f = (12, 8)\ (12, -8)$

Hence f is not a function as 12 shows two different images.